MTH241 Fall 2024: Quiz 02

Name: Samudai Champloo
UID:

Closed book, no calculator, show your work clearly.

1. (5pt) Let $P = (-3, 2, -1)$, $Q = (-7, 3, -4)$ and $R = (-5, -1, 0)$. Find the area A of the triangle PQR. (Grading: **2pt**: working; **2pt**: cross product; **1pt**: area)

$PQ = Q - P = (-4, 1, -3)$

$PR = R - P = (-2, -3, 1)$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -4 & 1 & -3 \\ -2 & -3 & 1 \end{vmatrix} = \begin{bmatrix} -8 \\ 10 \\ 14 \end{bmatrix}$$

$$A = \frac{1}{2}\|PQ \times PR\| = \frac{1}{2}\sqrt{8^2 + 10^2 + 14^2}$$ (no need to simplify)

2. (5pt) Let the point $(1, -2, 5)$ and the plane be defined as $3(x-1) - 4(y+2) + 12z = 0$. Show tht the point is not on the plane and find the distance D from the point to the plane. (Grading: **1pt**: working; **2pt**: proof; **2pt**: distance)

$P = (1, -2, 5)$

$E: 3(x-1) - 4(y+2) + 12z = 0$

$Q = (1, -2, 0)$ pt on E

$N = (3, -4, 12)$ normal vector

$PQ = (0, 0, -5)$

$3 \cdot (1-1) - 4(-2+2) + 12 \cdot 5 = 60 \neq 0$

so P not on plane.

$$\text{dist} = \frac{|PQ \cdot N|}{\|N\|} = \frac{\left|\begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -4 \\ 12 \end{bmatrix}\right|}{\|N\|} = \frac{|-60|}{\sqrt{3^2 + 4^2 + 12^2}} = \frac{60}{\sqrt{9 + 16 + 144}}$$ (no need to simplify)